PERGAMON

International Journal of Multiphase Flow 27 (2001) 1051–1063

International Journal of Multiphase Flow

www.elsevier.com/locate/ijmulflow

Brief communication

An Eulerian turbulent diffusion model for particles and bubbles

E. Loth *

Department of Aeronautical and Astronautical Engineering, University of Illinois, Urbana, IL, USA

Received 17 December 1999; received in revised form 23 September 2000

1. Background

1.1. Turbulent particle dispersion

Turbulent dispersion of particles and droplets in free shear flows is important in many two-phase flow areas, such as sprays and particle combustion or bubble diffusion in rising plumes. Turbulent dispersion for two-phase flows herein refers to the evolution of particle, droplet, or bubble concentration due to the underlying continuous-phase turbulence. In general, turbulent dispersion can be separated into two different aspects: (a) *mean diffusion*, which characterizes only the overall mean (time-averaged) spread rate of particles caused by the mean statistical properties of the turbulence, and (b) *structural dispersion*, which includes the details of the non-uniform particle concentration structures generated by local instantaneous features of the flow, primarily caused by the spatio-temporal turbulent eddy features and evolutions. The differences between these two aspects are shown in Fig. 1 for example the case of particle diffusion in a boundary layer.

If one uses direct numerical simulation (DNS) to resolve the turbulent eddies down to the Kolmogorov lengths, then the two aspects are obtained directly. Similarly, large eddy simulation (LES) can be used to obtain at least part of the structural dispersion features. Unfortunately for most engineering problems for turbulent flows of high Reynolds number, the computational resources for DNS or LES are far beyond those presently available. As such, the most common approach is to use Reynolds averaged Navier–Strokes (RANS) for the continuous-fluid which are based on empirical closure arguments for predicting mean turbulent momentum diffusion. This can be illustrated by the trends shown in Fig. 2 (Loth, 2000) where the number of computational nodes and the total CPU (iterations times number of nodes) becomes very high at large Reynolds

* Fax: +217-244-5581.
E-mail address: e-loth@uiuc.edu (E. Loth).

0301-9322/01/$ - see front matter © 2001 Elsevier Science Ltd. All rights reserved.
PII: S0301-9322(00)00057-4

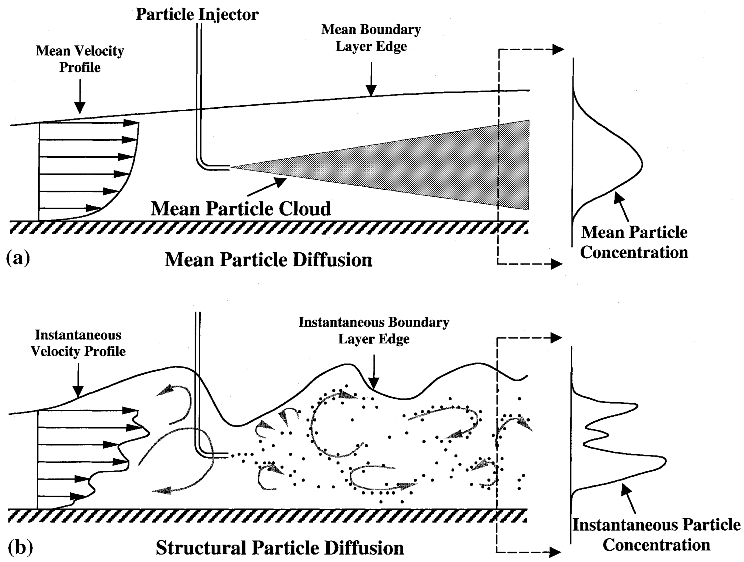

Fig. 1. Particle distribution in a turbulent boundary layer showing: (a) mean diffusion and (b) structural dispersion.

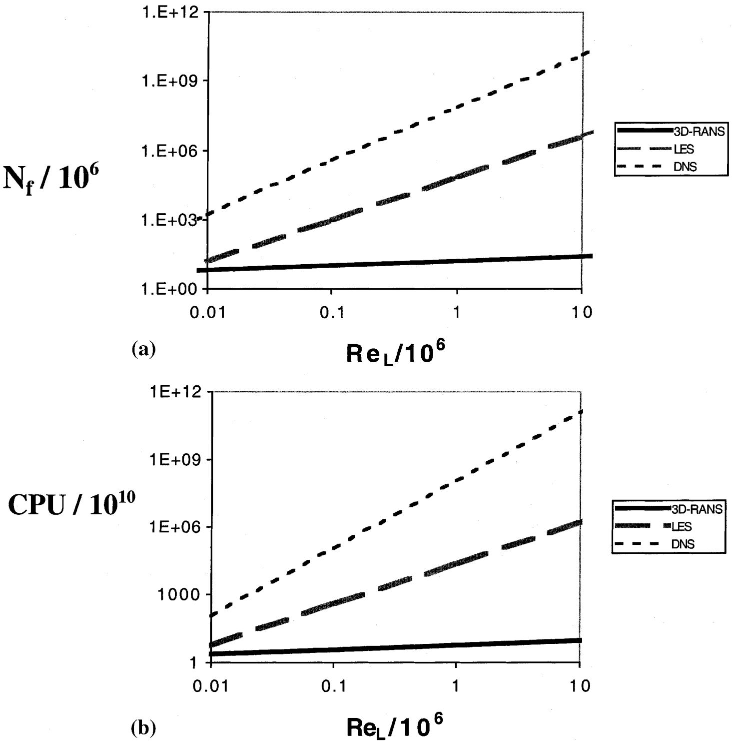

Fig. 2. Approximate requirements to compute continuous-phase wall-bounded flows for: (a) number of nodes (N_f) and (b) CPU time ($N_f \times$ cycles).

numbers when viscous sub-layers need to be simulated. However for RANS only the mean particle diffusion aspect can be obtained, regardless of whether Lagrangian or Eulerian methods are used for the particle-phase. For this approach, robust models for particle diffusion can be of service for engineering calculations.

For mean diffusion, one assumes that the concentration distribution of the particles is smoothly varying in space on length scales associated with the mean flow features (not the instantaneous turbulent eddy features). To consider the particle density as a continuum in the mean (as in Fig. 1(a)), one can define the local particle number-density (n_p), which is the number of particles (N) per unit volume of the mixed-fluid (Ω_m) in the limit $n_p = \delta N/\delta\Omega_m$. For a continuum of n_p, the mixed-fluid volume contains many particles, i.e., $\delta\Omega_m \gg d^3/\alpha$, d is the particle diameter (or equivalent volumetric diameter if non-spherical), and α is the volume fraction (volume occupied by particles divided by the total mixed-fluid volume). For time-averaged turbulence, one must also average over the eddy integral scales (Λ), i.e., $\delta\Omega_m > \Lambda^3$.

To quantify mean diffusion, it is helpful to define the turbulent particle diffusion ratio, $\bar{D}$, which is defined as the ratio of mean particle diffusion to continuous-fluid turbulent diffusion of a scalar: $\bar{D} = \nu_{pt}/\nu_{st}$. The turbulent particle Schmidt number is defined as the ratio of continuous-fluid momentum diffusion to particle diffusion in turbulence: $\mathrm{Sc}_p = \nu_{ft}/\nu_{pt}$ where ν_{ft} is the turbulent eddy viscosity of the fluid. This is analogous to the turbulent Schmidt number which is the ratio of momentum diffusion to mass (scalar) diffusion: $\mathrm{Sc}_{ft} = \nu_{ft}/\nu_{st}$. Typically, mass diffuses faster than momentum in turbulent free shear flows and Sc_{ft} is generally reported as 0.7 for jets and mixing layers (Crowe et al., 1988; Faeth, 1987; Yakhot and Orzag, 1986); Sc_p is equal to Sc_{ft} for the case of infinitely small particles in such flows. For finite size particles, Csanady (1963) notes that mean particle diffusion varies quadratically for short-time periods ($t \ll \tau_\infty$) but linearly for long-time periods ($t > \tau_\infty$), where τ_∞ is the long-time diffusivity time-scale and significantly exceeds both τ_p (the particle response time) and τ_{int} (the particle-eddy interaction time-scale). The latter criterion is demonstrated by the computational results of Elghobashi and Truesdell (1992) and Bocksell and Loth (1998), which showed that long-time diffusion was not obtained until the integration time had substantially exceeded the particle response times and interaction times by as much as a factor of eight, i.e., $\tau_\infty \gg \max(\tau_p, \tau_{int})$.

1.2. Four features of mean turbulent particle diffusion

For heavy particles which are dependent primarily on drag and gravity, the mean diffusion can be characterized by four features described in the following. Herein the surrounding fluid is assumed to behave in a continuum and the particle concentration is dilute, such that particle–particle interactions and two-way coupling effects can be ignored. Criteria for this assumption can be found in Elghobashi (1994), Crowe et al. (1998) and Loth (2000).

The most often cited mean diffusion feature is the dependence of Stokes number for free shear flows at constant gravity and continuous-flow conditions. The Stokes number is conventionally defined as the ratio of the particle kinematic response time to the local continuous-fluid turbulent integral time scale: $\mathrm{St}_\Lambda = \tau_p/\tau_\Lambda$. The particle response time for a sphere can be written as a function of particle diameter (d) and the ratio of particle density to continuous-fluid density ($\Psi = \rho_p/\rho_f$) as

$$\tau_p = \rho_p(1 + C_M/\Psi)d^2/(18\mu_f f_{term}), \quad (1)$$

where C_M is the added mass coefficient and f_{term} is the Stokes correction to the drag coefficient at terminal velocity (V_{term}). The general Stokes correction (f) is defined as the ratio of the particle drag coefficient (C_D) to the Stokes drag coefficient, i.e., $f = (C_D Re_p)/24$, and is approximately equal to unity for particle Reynolds number less than unity. The Reynolds number is in turn based on the magnitude of the relative velocity : $Re_p = \rho_f|\mathbf{V}_{rel}|d/\mu_f$, where the relative velocity vector is the difference between the particle velocity and the fluid velocity immediately surrounding the particle, i.e., $\mathbf{V}_{rel} = \mathbf{V}_p - \mathbf{V}_{f,p}$.

The primary significance of the Stokes number is that it dictates how readily a particle can follow the fluctuations of an eddy. For Stokes numbers much less than unity, the particle will act nearly as a passive tracer since it quickly responds to the continuous-fluid fluctuations, i.e., $\bar{D} \sim 1$. For Stokes numbers much greater than unity and finite gravity, the particle will not be able to respond to the fluctuations yielding a trajectory that is primarily controlled by mean convection and gravity i.e., $\bar{D} \sim 0$. For intermediate Stokes numbers, the diffusion is typically bounded by these extremes and monotonically decreases as particle diameter increases (for fixed gravity). However, there have been reports of mean particle diffusion rates in excess of that for a scalar ($\bar{D} > 1$) in jets and planar shear layers with particles for Stokes numbers of order unity (Crowe, 1982). This effect has been referred to as "diffusion peaking", but unfortunately most of the experimental data which have demonstrated this feature contain significant uncertainties rendering the results primarily qualitative (see Loth, 1998). In addition, the experiments were for non-homogeneous flows with $t < \tau_\infty$, such that long-time diffusion may not have been reached. Substantial diffusion peaking has not been experimentally observed for fully developed homogeneous isotropic turbulence with $t > \tau_\infty$.

The second particle diffusion feature is the crossing-trajectory effect, which results when the particle-eddy interactions are dominated by a particle traversal of the eddy (as opposed to the eddy lifetime). This effect is related to the drift parameter which is defined as the ratio of the particle terminal velocity to the turbulent fluctuation intensities, i.e., $\gamma = V_{term}/V'_{f,rms}$ for isotropic turbulence. Wells and Stock (1983) have shown that this effect decreases diffusion as γ increases for constant Stokes numbers. This crossing-trajectory effect can also be interpreted as a mean diffusion dependence on eddy Froude number, Fr_Λ, as discussed by Reeks (1977). The eddy Froude number is defined as the ratio of hydrostatic to integral-scale hydrodynamic pressure gradients given as $Fr_\Lambda = V'_{f,rms}2/4g\Lambda$ (Taeibi-Rahni et al., 1994). Since we can write $\tau_p = (\Psi + C_M)(V_{term})/(|\Psi - 1|g)$ and $\tau_\Lambda = \Lambda/V'_{f,rms}$ for an isotropic turbulent flow, then $St_\Lambda = (4Fr_\Lambda\gamma)(\Psi + C_M)/|\Psi - 1|$. Thus for a given value of Ψ and C_M, we note that of the three integral-scale non-dimensional parameters $(St_\Lambda, \gamma, Fr_\Lambda)$, only two are independent. This conclusion is consistent with St_Λ and γ identified as the primary parameters in several recent analytical studies for heavy particle diffusion (Stock, 1996; Mei and Adrian, 1993; Hunt et al., 1994) as well as St_Λ and Fr_Λ identified for buoyant particle diffusion (Sene et al., 1994; Gañán-Calvo and Lasheras, 1991; Loth, 1998). Note that the above is limited to conditions where particle shape effects, non-linear drag-effects, etc., can be linearly included into the Stokes correction factor (used in the definition of particle response time). For particles no longer governed by $\Psi \gg 1, Re_p$ may also become important if there is mean diffusion due to viscous lift fluctuations.

The third feature of mean particle diffusion is the inertial-limit behavior, which is the long-time diffusion of particles with very large response times in negligible gravity ($St_\Lambda \gg 1$ and $\gamma \ll 1$). For isotropic homogeneous flow, Reeks (1977) and Stock (1996) note that this limit yields finite (non-zero) diffusion. The long-time $\bar{D}$ in this limit is essentially proportional to the ratio of the Lagrangian to the moving-Eulerian continuous fluid time correlation time-scales ($\tau_{\Lambda \mathrm{L}}/\tau_{\Lambda \mathrm{mE}}$). If the two time scales are approximately equal, the inertial-limit behavior yields a mean diffusion about that of a passive scalar, i.e., $\bar{D} \sim 1$. However, the relationship between these two time-scales has been the subject of recent debate because of some conflicting results.

With respect to experiments, Wells and Stock (1983) measured particle diffusion for zerogravity wake-turbulence and found only a small increase in diffusion (within experimental uncertainty), suggesting $\tau_{\Lambda \mathrm{L}}/\tau_{\Lambda \mathrm{mE}} \sim 1$. In addition, cinematic Particle Image Velocimetry experiments by Loth and Stedl (1999) measured two-dimensional versions of $\tau_{\Lambda \mathrm{L}}$ and $\tau_{\Lambda \mathrm{mE}}$ in a high Reynolds number shear layer and also found they were nearly equal when based on velocity fluctuations. However, Sato and Yamamoto (1987) measured $\tau_{\Lambda \mathrm{L}}$ and compared these to induced values of $\tau_{\Lambda \mathrm{mE}}$ and found $\tau_{\Lambda \mathrm{L}}/\tau_{\Lambda \mathrm{mE}}$ in the range of 0.3–0.6 depending on the flow Reynolds number. With respect to simulations, Wang and Stock (1993) approximated turbulence with a Fourier series simulations and found $\tau_{\Lambda \mathrm{L}}/\tau_{\Lambda \mathrm{mE}}$ should be much less than unity, e.g., 0.4. However, isotropic DNS simulations of decaying turbulence by Elghobashi and Truesdell (1992) and of stationary turbulence by Yueng and Pope (1989) and Groszmann et al. (1999) demonstrated that $\tau_{\Lambda \mathrm{L}}/\tau_{\Lambda \mathrm{mE}} \sim 1$. The Elghobashi and Truesdell (1992) study also noted that the Lagrangian correlation was somewhat greater for short-times ($t < \tau_{\Lambda \mathrm{L}}$) and somewhat smaller for long-times, which may explain some of the discrepancies. Based on the above results, a time-scale ratio of approximately unity is suggested herein, and is consistent with the suggestion of Viollet and Simonin (1994). However, further research is obviously needed to quantify this time-scale ratio.

The fourth feature of mean particle diffusion is the continuity effect, which arises due to the negative loops in the lateral correlation for an exponentially decaying longitudinal correlation function when moving relative to the turbulent fluctuations. This results in the velocity correlation function yielding lateral correlation length scales that are asymptotically one-half the longitudinal correlation length scales (Csanady, 1963). This ratio has been confirmed by experiments and simulations (Stock, 1996, Elghobashi and Truesdell, 1992). Stock (1996) notes that the continuity effect essentially modifies the eddy lifetime seen by the particle depending on whether the diffusion considered is streamwise or transverse to the mean particle direction.

To summarize, the mean diffusion of heavy particles (dependent primarily on drag and gravity) in homogeneous isotropic turbulence contains four fundamental features:

(a) Stokes number effect (reduced diffusion for larger particles in finite gravity);
(b) crossing-trajectory effect (reduced diffusion for increasing γ);
(c) inertial-limit behavior (proper limit of finite diffusion for infinite St_Λ and $\gamma = 0$);
(d) continuity effect (anisotropic diffusion between longitudinal and lateral directions).

1.3. Lagrangian modeling of the mean particle diffusion

Stochastic Lagrangian models have had significant success in describing turbulent diffusion and they are overviewed herein since their basic timescales representations are employed for the present Eulerian diffusion model. These Lagrangian formulations employ stochastic sampling to simulate the turbulent fluctuations of the fluid at the particle location. In the most common case, the time-averaged velocities and mean turbulence correlations from Eulerian RANS computations of the continuous phase are used as the statistical correlations to construct the stochastic sampling (MacInnes and Bracco, 1992). A particularly successful technique is the continuous random walk (CRW) model which has been shown to quantitatively describe particle diffusion in a variety of free shear flows (e.g., Bocksell and Loth, 1998). This technique uses randomly generated continuous-fluid velocity fluctuations at each timestep coupled with a finite correlation between timesteps. With anisotropic flows, the magnitude of the random velocity fluctuations is specified by the mean velocity stress tensor $\overline{u'_{fj}u'_{fj}}$. If one assumes isotropy, then the magnitude is based on overall turbulent kinetic energy (k).

These stochastic models require timescales to quantify the statistical decorrelation of the velocity perturbations. The particle-eddy interaction an interaction time scale is often given as

$$\tau_{\text{int},ij}^{-2} = \tau_{\Lambda,ij}^{-2} + \tau_{\text{tra},ij}^{-2} \tag{2}$$

The isotropic eddy lifetime is typically given as $\tau_\Lambda = c_\tau k/\varepsilon$, where ε is turbulent dissipation, and c_τ is an empirical constant, ca. 0.27, which is based on experimental data for two-phase particle dispersion (MacInnes and Bracco, 1992, Bocksell and Loth, 1998).

The eddy traversal time of (2) for isotropic turbulence can be estimated from the relative velocity and the eddy integral scale as $\tau_{\text{tra},i} = V_{\text{rel}}/\Lambda_i$. The local integral eddy size is given by $\Lambda_i = c_\Lambda c_{c,i} c_\mu^{3/4} k^{3/2}/\varepsilon$, where c_μ is the conventional RANS turbulent length-scale coefficient ($=0.09$), and c_Λ is again an empirical coefficient obtained through calibration with experimental data, ca. 1.6 (Bocksell and Loth, 1999). Note that the ratio c_τ/c_Λ is proportional to the structure parameter (Stock, 1996) and is herein assumed a constant for free-shear flows. However, it can vary for non-equilibrium turbulent flows (Hunt et al., 1994; Stock, 1996; Graham, 1998) particularly in the case of wall-bounded flows (Kallio and Reeks, 1989; Ushijima and Perkins, 1999). The continuity effect is incorporated by using $c_{c,i} = 1 + V_{\text{rel},}i/|\mathbf{V}_{\text{rel}}|$ as described by Bocksell and Loth (1999) where i is the streamwise direction of the particle (along $\mathbf{V}_\text{p}$). After a time exceeding several τ_p's, the streamwise direction will correspond to the gravity direction, in which the above expression is consistent with that of Graham (1998).

Application of the above timescales with the GRW Lagrangian technique for various free shear flows (wakes and jets) yields high fidelity with respect to mean particle diffusion for integration times both less than and greater than the long-time diffusivity time scale, τ_∞ (Bocksell and Loth, 1998). Results for simple isotropic homogeneous turbulence for heavy particles ($\Psi \gg 1$) with linear (Stokesian) drag coefficients were also obtained with the CRW model. Fig. 3 from Bocksell and Loth (1998) describes the change in D_{11} (transverse diffusion rates) as St_Λ and Fr_Λ vary using a CRW model for homogeneous isotropic turbulence. The results show that the diffusion monotonically reduces as St_Λ increases, and this reduction is delayed as the Froude number increases. As the Froude number approaches infinity (zero-gravity), the reduction is indefinitely delayed, thus yielding the inertial limit feature. Inclusion of fluctuating lift and fluid-stress forces for buoyant particles (bubbles) in the same flow yielded qualitatively similar results, indicating that the drag forces still dominate. This is consistent with the successful bubbly jet flow predictions of Sun and Faeth (1986); although Michaelidis (1997) showed that in low particle Reynolds number circumstances Basset history can significantly contribute to turbulent diffusion.

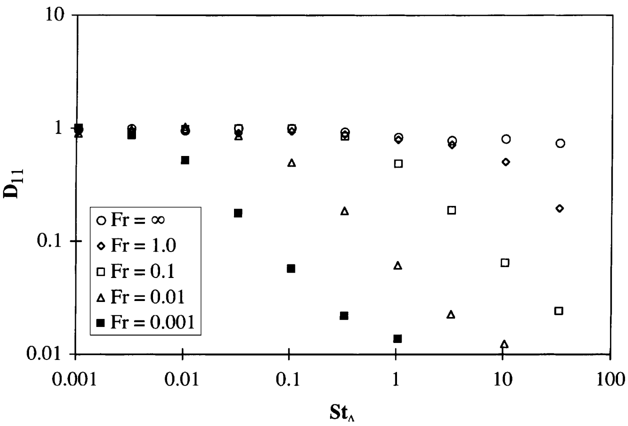

Fig. 3. Transverse particle diffusion ratio as a function of particle Stokes number and eddy Froude number from Lagrangian simulations.

2. Eulerian modeling of mean particle diffusion

For simulating turbulent diffusion of particles, the Eulerian approach has several distinct advantages over Lagrangian methods. For particles present throughout a large extent of the computational domain, the Eulerian description typically requires fewer degrees of freedom to quantitatively describe particle concentration variations, since statistical collection is needed for the Lagrangian case (especially if a stochastic diffusion model is used). Lagrangian methods can also be problematic if the particle-cloud volume is larger than the fluid-averaging volume and/or is difficult to compute (Loth, 2000). In addition, an Eulerian approach allows both phases to be handled with a consistent numerical scheme and a consistent numerical grid. This discretization coincidence for an Eulerian treatment of the particles becomes a distinct accuracy advantage when one is trying to compute the effects of the particles on the continuous fluid for two-way coupling (Shrayber, 1979, Sivier et al., 1996). In the following, we are interested in predicting mean turbulent diffusion within the Eulerian framework while retaining the turbulent diffusion performance of the Lagrangian CRW approaches.

The time-averaged or density-averaged RANS equations for the dispersed-phase involve correlations of mean particle spatial density ($\bar{\sigma}$) and velocity ($\overline{u_{pi}}$) to describe the mass diffusion due to turbulence. For example, the time-averaged continuity equation can be written as:

$$\partial\bar{\sigma}/\partial t + \partial(\bar{\sigma}\overline{u_{pj}})/\partial x_j = -\partial(\overline{\sigma' u'_{pi}})/\partial x_j$$

Use of a particle-eddy-viscosity model (which essentially invokes a Fickian diffusion assumption) thus requires an Eulerian model for $\bar{D}$, i.e.,

$$-\overline{\sigma' u'_{pi}} = (\nu_{pt})(\partial\bar{\sigma}/\partial x_i) = (\nu_{ft}\bar{D}/Sc_{ft})(\partial\bar{\sigma}/\partial x_i)$$

This gradient-based diffusion assumption for long-time particle concentration is consistent with that for turbulent diffusion of momentum and mass for RANS treatments of the continuous-fluid. For the latter, one assumes that the length and time scales of the turbulence (Λ and τ_Λ)

are small compared to the characteristic features of the flow for which variations are to be predicted. Similarly, predicted variations of particle concentration are only valid over convected time scales of about τ_∞ or more. This is to insure sufficient temporal averaging of particle velocity variations and results in limit of applicability of such a model to convected lengths of $U_p\tau_\infty$ or more.

For the Favre-averaged approach, there is no source-term for the continuity equation but the momentum equation still requires an Eulerian model for $\bar{D}$ (Loth, 2000). Some studies simply use $\bar{D} = 1$ (e.g., Kashiwa and Gore, 1987) or $\bar{D} = 0$ (e.g., Vaidya et al., 1995), which are respectively only reasonable for particles which are sufficiently small ($St_\Lambda \ll 1$) or sufficiently large ($\gamma \gg 1$). Several algebraic models include a Stokes number dependence, e.g., models cited by Wu and Liu (1991) where $\bar{D} = 1/[1 + St_\Lambda^2]$ and by Shirolkar et al., 1996 where $\bar{D} = 1/[1 + St_\Lambda]$, but such expressions do not include the Fr_Λ dependence shown in Fig. 3. Similarly, other algebraic models noted by Alonso (1981), Faeth (1987), Soo (1990) and Loth (1998) do not incorporate inertial-limit behavior or the continuity effect and thus may not be as physically robust as the Lagrangian models.

An Eulerian diffusion model which does account for the four key diffusion features is given by Reeks (1977) as integro-differential equations. A more convenient algebraic expression was developed from the Reeks by Wang and Stock (1993). This model gives the transverse ($i = 1, 2$) and streamwise ($i = 3$) long-time diffusivity ($\bar{D}_{11}$ and $\bar{D}_{33}$) for homogeneous isotropic turbulence and assumes $\tau_{\Lambda L}/\tau_{\Lambda mE\sim}0.4$ (Stock, 1996). If one "modifies" their model by assuming $\tau_{\Lambda L}/\tau_{\Lambda mE\sim}1$ as was suggested in Section 1, the resulting diffusion ratios with a structure parameter of unity are:

Modified stock model:

$$\bar{D}_{11} = \bar{D}_{22} = [(\gamma^2 + 1)^{1/2} - (\gamma/2)]/[\gamma^2 + 1],$$
$$\bar{D}_{33} = [(\gamma^2 + 1)^{1/2}]/[\gamma^2 + 1]. \tag{3}$$

The present diffusion model assumes $\bar{D}_{ii} = \tau_{\mathrm{int}}/\tau_{\Lambda,i}$ as noted by Csanady (1963) and Viollet and Simonin (1994). This approximation is combined with (2) and the time scales from the above Lagrangian models (for particles which are drag-dominated) to give the anisotropic model

Present (Csandy-based) model:

$$\bar{D}_{\mathrm{ii}} = [1 + (2/3)(\gamma c_\tau)^2 (c_\Lambda c_{\mathrm{c},i} c_\mu^{3/4})^{-2}]^{-1/2},$$
$$\gamma = V_{\mathrm{rel}}/(2k/3)^{1/2}, \tag{4}$$
$$c_{\mathrm{c},i} = 1 + V_{\mathrm{rel,i}}/|\mathbf{V}_{\mathrm{rel}}|,$$

where the empirical coefficients are again $c_\tau = 0.27, c_\Lambda = 1.6$ and $c_\mu = 0.09$. The anisotropic version of this model becomes

Present anisotropic model:

$$\bar{D}_{ij} = [1 + (2/3)(\gamma_{ij} c_\tau)^2 (c_\Lambda c_{c,i} c_\mu^{3/4})^{-2}]^{-1/2},$$
$$\gamma_{ij} = V_{\mathrm{rel}}/[\overline{u'_{\mathrm{f}i} u'_{\mathrm{f}j}}]. \tag{5}$$

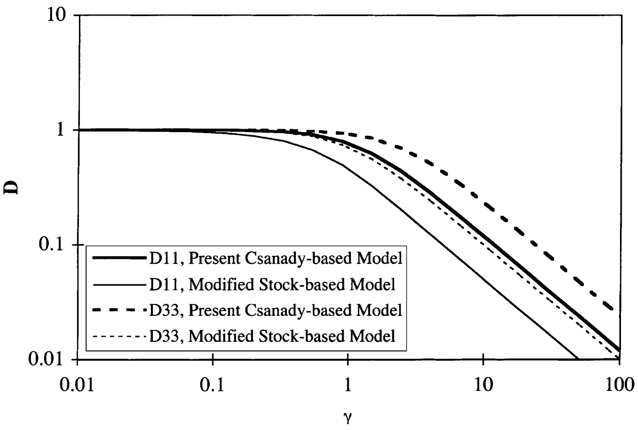

Fig. 4. Eulerian models of particle diffusion ratio as a function of γ (ratio of particle terminal velocity to fluid fluctuations).

In both the Stock model and present Eulerian model, the analytical description provides a crossing-trajectory effect ($\bar{D}$ decreases as γ increases), a continuity effect ($\bar{D}_{11}$ becomes one half of $\bar{D}_{33}$ for very large γ values), and an inertial-limit effect (non-zero $\bar{D}$ as St_A approaches infinity for $\gamma = 0$). The present model and the "modified" Stock model are shown in Fig. 4 for isotropic turbulence, where it is clear that both models yield a linear decrease in particle diffusion at large γ, such that the differences between the two models are only quantitative. In fact, the modified Stock model can be shown to have nearly the same quantitative behavior as the present model by adjustment of their structure parameter.

Now let us consider the fidelity of the present model for long-time diffusion of particle and bubbles in nearly homogeneous isotropic turbulence based on experimental, DNS, and CRW results. Fig. 5 shows the transverse diffusion of the present (Csanady-based) Eulerian model as a function of γ. We observe that the heavy and buoyant particle CRW Lagrangian stochastic simulation results (for a variety of Stokes numbers) approximately match the trends of the present Eulerian diffusion model. In addition, there is reasonable agreement with all the data for homogeneous isotropic turbulence: the DNS results of heavy particles by Elghobashi and Truesdell (1992), the measurements of heavy particles by Wells and Stock (1983), and the measurements of buoyant particles by Lasheras (1998) and Poorte (1998). The results indicate that mean particle diffusion is dominated by γ and not St_A, i.e., the variations due to Fr_A when the CRW results were plotted vs St_A (Fig. 3) are eliminated when plotted vs γ (Fig. 5). This is consistent with the experiments by Wells and Stock (1983) and DNS simulations by Groszmann et al. (1999). It also suggests that the present assumption of $\tau_{AL}/\tau_{AmE} \sim 1$ is reasonable. In addition, CRW simulations by Bocksell and Loth (1998, 1999) noted that use of non-linear drag coefficients did not significantly modify these long-time diffusion results.

The agreement of the present model with buoyant particle results indicates that drag-based mean diffusion typically dominates over effects caused by stress-gradient and lift fluctuations. The result that the bubble diffusion is dominated by the velocity ratio (γ) is consistent with that suggested by non-homogeneous flow simulations of Tio et al. (1993) for a frozen vortex field and by Sene et al. (1994) for a two-dimensional point-vortex shear layer. This dominance was also

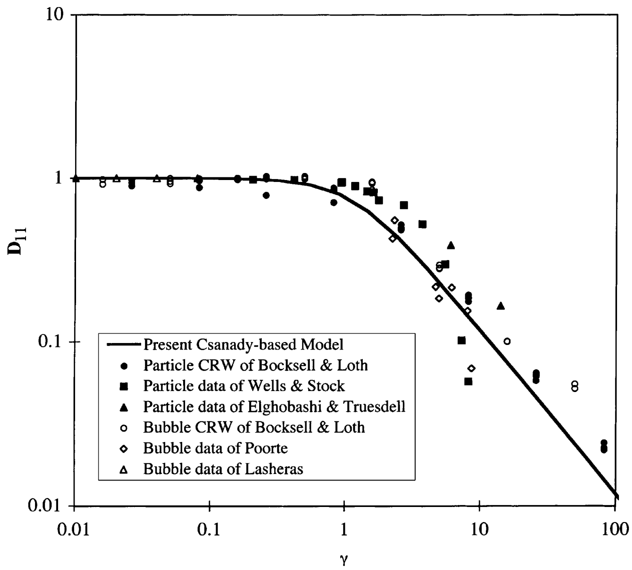

Fig. 5. Transverse particle diffusion ratio as a function of γ (ratio of particle terminal velocity to fluid fluctuations).

shown in recent experiments by Felton (1999) for spherical bubbles in a turbulent boundary layer and by Ford and Loth (2000) for ellipsoidal bubbles in a free shear layer. However, some of the variation may be due to the importance of secondary forces, e.g., diffusion due to fluctuations in lift, fluid-stress, and Basset history forces. In addition, the predictions of Sene et al. (1994) and Hunt et al. (1994) suggest that inhomogeneity may cause significant skewing of the bubble concentration profile and increase the importance of the eddy Froude number.

Notably, the present Eulerian models for $\bar{D}$ does not predict the "diffusion peaking" qualitatively noted for some jet flows. However, the computational results of Elghobashi and Truesdell (1992) suggest that this peaking behavior is at least partly due to the fact that heavy particles disperse more than fluid particles for integration times less than τ_∞. In fact, the long-time diffusion limit is often not achieved in many engineering two-phase flows, e.g., much of the diffusion reported in Snyder and Lumley (1971); whereas, the above Eulerian models are only appropriate for long-time diffusion conditions, i.e., convection times greater than τ_∞. Note that Lagrangian approaches do not generally suffer from this long-time limitation and are able to predict this peaking using resolved-eddy simulations (e.g., Crowe, 1982) and even with CRW simulations (Bocksell and Loth, 1998).

Finally, the present diffusion model can also be used as a subgrid particle diffusion model for Eulerian Large Eddy Simulations of the dispersed-phase by employing a particle-eddy-viscosity. The particle subgrid diffusion is simply the product of the continuous-phase subgrid viscosity and the subgrid particle diffusion ratio: $\nu_{p\Delta} = \nu_{s\Delta}\bar{D}_\Delta$. In the Favre-averaged dispersed-phase LES equations (Loth, 2000), one may then compute $\bar{D}_\Delta$ using the above Csanady-based diffusion model by substituting the sub-grid drift parameter, $\gamma_\Delta = V_{\text{term}}/(2k_\Delta/3)^{1/2}$ for γ in (4), where the subgrid turbulent kinetic energy (k_Δ) can in turn be taken as linearly proportional to the trace of sub-grid stress-tensor. Of course, if the particles are sufficiently large ($\gamma_\Delta \gg 1$) we may employ $\bar{D}_\Delta = 0$, and if the particles are sufficiently small ($\gamma_\Delta \ll 1$) we may employ $\bar{D}_\Delta = 1$.

3. Conclusions

A simple Eulerian (Csanady-based) model was constructed for long-time diffusion of heavy or bouyant particles and incorporates the diffusion four features: Stokes number, crossing-trajectory, inertial limit, and continuity effects. The model was found to it provide reasonable agreement for long-time diffusion in homogeneous turbulence based on experimental, DNS, and CRW results. A key advantage of the proposed model is that it yields a closed-form analytical result similar to Stock (1996). However, a more complex model can be developed to include secondary particle forces, short-term diffusion, and more detailed representations of the turbulence field.

Acknowledgements

This work was supported by the Office of Naval Research under grant N00014-96-1-0312. Mr. Todd Bocksell also provided valuable comments.

References

Alonso, C.V., 1981. Stochastic models of suspended-sediment dispersion. ASCE J. Hydraul. 107, 733–757.

Bocksell, T., Loth, E., 1998. Stochastic diffusion models for particles in wakes and jets. In: ASME Summer Fluids Engineering Meeting, FEDSM98-5017, Washington, DC, June.

Bocksell, T., Loth, E., 1999. A CRW model for diffusion of bubbles and particles. In: ASME Summer Fluids Engineering Meeting, San Francisco, June, FEDSM99-7366.

Crowe, C.T., Chung, J.N., Troutt, T.R., 1988. Particle mixing in free shear slows. Prog. Energy Combust. Sci. 14, 171–194.

Crowe, C.T., 1982. REVIEW – Numerical models for dilute gas-particle flows. Fluid Eng. 104, 297–303.

Crowe, C.T., Sommerfeld, M., Tsuji, Y., 1998. Multiphase Flows with Droplets and Particles. CRC Press, Boca Raton, FL.

Csanady, G.T., 1963. Turbulent diffusion of heavy particles in the atmosphere. J. Atmos. Sci. 20, 201–208.

Elghobashi, S., 1994. On predicting particle-laden turbulent flows. Appl. Sci. Res. 52, 309–329.

Elghobashi, S., Truesdell, G.C., 1992. Direct simulation of particle dispersion in a decaying isotropic turbulence. J. Fluid Mech. 242, 655–700.

Faeth, G.M., 1987. Mixing, transport and combustion in sprays. Prog. Energy Combust. Sci. 13, 293–345.

Felton, K., 1999. Sub-millimetric bubbly flow in a vertical turbulent boundary layer. Ph.D. Thesis, Mechanical Engineering Department, University of Illinois, Urbana-Champaign, 1999.

Ford, B., Loth, E., 2000. Diffusion of ellipsoidal bubbles in a turbulent shear layer. Int. J. Multiphase Flow 26, 503–516.

Gañán-Calvo, A.M., Lasheras, J.C., 1991. The dynamics and mixing of small spherical particles in plane, free shear layer. Phys. Fluids 3, 1207–1217.

Graham, D.I., 1998. Improved eddy interaction models with random length and time scales. Int. J. Multiphase Flow 24, 335–345.

Groszmann, D.E., Fallon, T.M., Rogers, C.B., 1999. Decoupling the roles of inertia and gravity on the preferential concentration of particles. In: ASME Summer Fluids Engineering Meeting, San Francisco, FEDSM99-7871, June.

Hunt, J.C.R., Perkins, R.J., Fung, J.C.H., 1994. Problems in modeling disperse two-phase flows. Appl. Mech. Rev. 46, S49–S60.

Kashiwa, B.A., Gore, R.A., 1987. A four equation model for multiphase turbulent flow. In: Proceedings of the 1987 ASME–JSME Thermal Engineering Joint Conference, vol. 5. ASME, New York, p. 261–267.

Kallio, G.A., Reeks, M.W., 1989. A numerical simulation of particle deposition in turbulent boundary layers. Int. J. Multiphase Flow 15, 433–446.

Lasheras 1998. University of California, San-Diego, personal communication.

Loth, E., 1998. An Eulerian model for turbulent diffusion of particles in free shear layers. AIAA J. 36, 12–17.

Loth, E., 2000. Numerical approaches for dispersed particles, droplets, and bubbles. Prog. Energy Combust. Sci. 26, 161–223.

Loth, E., Stedl, J., 1999. Taylor and Lagrangian correlations in a turbulent free shear layer. Exp. Fluids 26, 1–6.

MacInnes, J.M., Bracco, F.V., 1992. Stochastic particle dispersion modeling and the tracer-particle limit. Phys. Fluids A 4, 2809–2824.

Mei, R., Adrian, R.J., 1993. Effect of turbulence structure on the dispersion of fluid and particles. Gas-Solid Flows, vol. 166. ASME Fluids Engineering Divisions, p. 131–143.

Michaelidis, E., 1997. Reveiw–The transient equation of motion for particles, bubbles, and droplets. AMSE J. Fluids Eng. 119, 233–247.

Poorte, R.E.G., 1998. On the Motion of Bubbles in Active Grid Generated Turbulent Flows. Ph. D. Thesis, University of Twente, Netherlands, September.

Reeks, M.W., 1977. On the dispersion of small particles suspended in an isotropic turbulent fluid. J. Fluid Mech. 83, 529–546.

Sato, Y., Yamamoto, K., 1987. Lagrangian measurement of fluid-particle motion in an isotropic turbulent field. J. Fluid Mech. 175, 183–199.

Sene, K.J., Hunt, J.C.R., Thomas, N.H., 1994. The role of coherent structures in bubble transport by turbulent shear flows. J. Fluid Mech. 259, 219–240.

Shirolkar, J.S., Coimbra, C.F.M., Quirez McQuary, M., 1996. Fundamental aspects of modeling turbulent particle dispersion in dilute flows. Prog. Energy Combust. Sci. 22, 363–399.

Shrayber, A.A., 1979. Euler and Lagrange methods in the theory of two-phase flows with variable particle size of the discrete phase. Fluid Mech. Sov. Res. 8, 79–87.

Sivier, S.A., Loth, E., Baum, J.D., 1996. Dusty shock flow with unstructured adaptive finite elements and parcels. AIAA J. 34, 1078–1080.

Snyder, W.H., Lumley, J.L., 1971. Some measurements of particle velocity autocorrelation in approximately isotropic turbulence. J. Fluid Mech. 48, 41.

Soo, S.L., 1990. Multiphase Fluid Dynamics. Gower Technical, Aldershot-Brookfield, USA.

Stock, D.E., 1996. Particle dispersion in flowing gases – 1994 Freeman Scholar Lecture. ASME J. Fluids Eng. 118, 4–17.

Sun, T.-Y., Faeth, G.M., 1986. Structure of turbulent bubbly jets. Int. J. Multiphase Flow 12, 115.

Taeibi-Rahni, M., Loth, E., Tryggvason, G., 1994. Flow modulation of a planar free shear layer with large bubbles – direct numerical simulations. Int. J. Multiphase Flow 20, 1109–1128.

Tio, K.-K., Liñán, A., Lasheras, J.C., Gañán-Calvo, A.M., 1993. The dynamics of bubbles in periodic vortex flows. Appl. Sci. Res. 51, 285–290.

Ushijima, T., Perkins, R.J., 1999. Evaluation of Co and To in Turbulent Pipe or Channel Flows. In: ASME Summer Fluids Engineering Meeting, FEDSM99–7759, San Francisco, July.

Vaidya, N., Dionne, P.J., Singhal, A.K., 1995. A two-fluid model finite volume formulation for computing two-phase flows in complex geometries. ASME FED, Gas Liquid Flows 225, 179–191.

Viollet, P.L., Simonin, O., 1994. Modeling Dispersed Two-Phase Flows: Closure, Validation and Software Development" part of Mechanics USA 1994, Appl. Mech. Rev. 47, June, S80–S84 (also ASME Reprint No. AMR146).

Wang, L.P., Stock, D.E., 1993. Dispersion of heavy particles by turbulent motion. J. Atmos. Sci. 50, 1897–1913.

Wells, M.R., Stock, D.E., 1983. The effects of crossing trajectories on the dispersion of particles in a turbulent flow. J. Fluid Mech. 136, 31–62.

Wu, B., Liu, D., 1991. Modeling of lateral particle dispersion in FFB. In: Cavitation and Multiphase Flow Forum ASME Fluids Engineering Division, vol. 109, p. 15–19.

Yakhot, V., Orzag, S.A., 1986. Renormalization group analysis for turbulence. I. Basic theory. J. Sci. Comput. 1 (1).
Yueng, P.K., Pope, S.B., 1989. Lagrangian statistics from direct numerical simulations of isotropic turbulence. J. Fluid Mech. 207, 531–586.